GENERALIZED MODULAR DYNAMIC MODEL OF EARTHMOVING MACHINES

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Abstract. This research substantiates the necessity of utilizing a universal generalized modular dynamic model for wheeled earth-moving and transport machines (EMTMs). The distinctive features of the developed model include: the capability to describe the system's motion in both 2D and 3D variants; the modular construction of the mathematical model, allowing for the variability of parameter selection for its components (transmission type, working body type, etc.). Another notable aspect of the model is the inclusion of equations describing the motion of all actuators and the coupling equations between them, as well as the power balance equations. The synthesis of the generalized dynamic model employs methods and principles of analytical dynamics. The analysis of the proposed generalized modular dynamic model of EMTMs has enabled the development of a methodology for creating machine behaviour scenarios on the work site during technological operations. A key feature of this methodology is the simultaneous control of multiple actuators and mechanisms. This approach facilitates the solution of optimization problems related to the adaptation of EMTMs to variable operating conditions.

Keywords: system, dynamics, model, earth-moving machine, working process.

Introduction

Modern trends in the development of construction and earthmoving equipment suggest improving its performance indicators based on: adaptation of machines to changing operating conditions; optimization of machine and machine complex behaviour during technological operations through the use of artificial intelligence; application of modern technologies for developing the processed environment, etc. All these approaches involve the analysis of various scenarios of machine behaviour on the construction site.

A modern construction machine, in particular an earth-moving and transporting machine (EMTM), represents a complex spatio-temporal continuum of a finite set of interconnected systems and mechanisms. The interactions of individual systems, both with each other and with the environment, determine the behaviour of EMTM on the work site during technological operations. One method that allows reducing the decision-making time for improving the efficiency indicators of EMTMs is the analysis of the results of predicting the parameters of their dynamic states during operation. To conduct such studies, it is necessary to use generalized dynamic models of EMTMs developed based on system dynamics. In the field of EMTM design, the problems associated with the synthesis of a comprehensive generalized dynamic model of the machine are not fully covered and require further research.

Materials and methods

EMTMs are typical representatives of working machines. Their primary purpose is to process and transform the developed environment, which includes soils, building materials, and rocks. The operational experience of EMTMs shows that a large part of their efficiency indicators directly depends on the loading regime. Therefore, the design methodology of EMTMs is significantly determined by the characteristics of the force impacts on the working equipment, running gear, and actuators [1; 2].

Modern EMTM design theory compiles the following main methodologies (Fig. 1):

- methodology for determining basic parameters based on quasi-static machine loading models;
- methodology for determining EMTM element parameters based on ultimate dynamic loads arising from intense variations in external influences. These models are deterministic in nature. Typically, situations of unsteady machine motion or its individual elements during forced acceleration or braking are considered. For example, deterministic dynamic models describing the interaction of the working equipment with the developed environment have become widespread for EMTMs: intense penetration of the working element into the developed material, as well as collision of the working element with an insurmountable rigid obstacle;
- methodology for determining EMTM parameters, taking into account the random nature of external influences. Here, three directions can be distinguished: models that account for the random nature of the initial conditions of the studied dynamic processes; models that account

for the probabilistic distribution of the physical and mechanical properties of the developed material; models that account for the random characteristics of the supporting surface (coefficient of adhesion, rolling resistance coefficient, random longitudinal profile of supporting surface irregularities, etc.). It should be noted that research considering the simultaneous effect of all three groups of random factors is extremely scarce and insufficiently studied;

- methodologies for fuzzy modeling of EMTM behaviour on the work site and the functioning of its individual systems and mechanisms;
- combined methodologies that combine various combinations of the above methodologies.

General approaches to the development and analysis of quasi-static, stochastic, and deterministic dynamic EMTM models are well studied and based on the classical analytical method algorithm. It is based on the following well-known principles (Fig. 1):

- separation of a complex object into individual components;
- identification of components;
- analysis of individual components;
- synthesis of a complex object from the studied components;
- formulation of final conclusions.

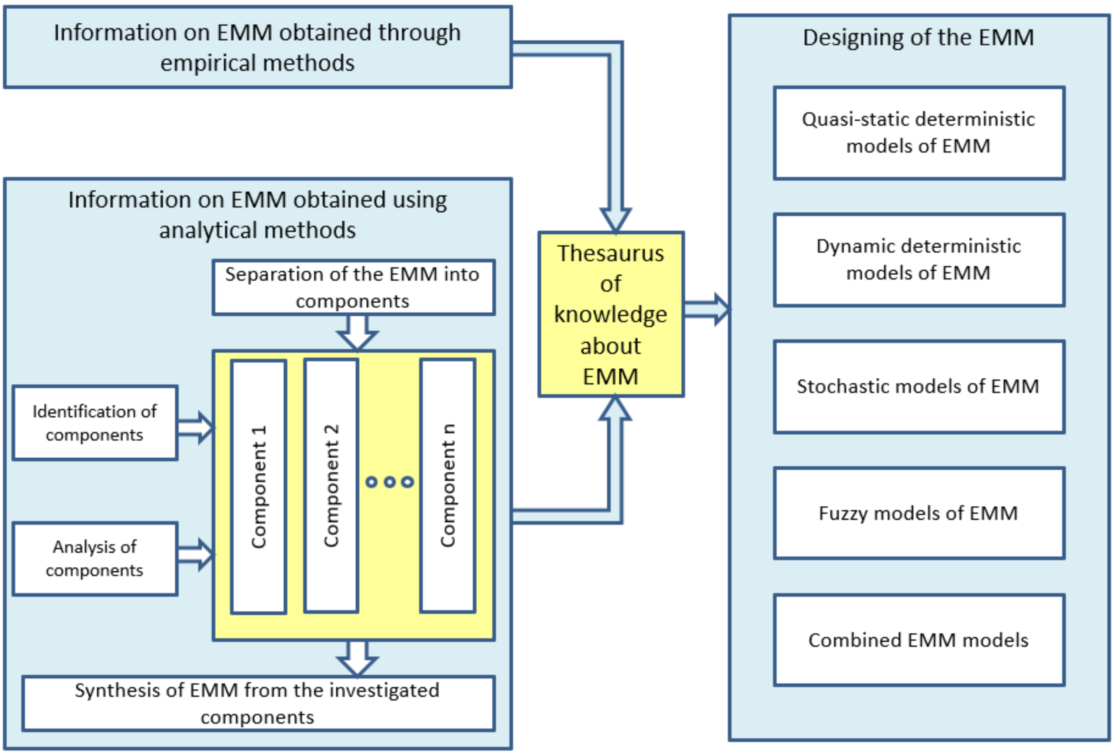

Fig. 1. **Structure of information exchange between EMTM research and design methodologies**

Synthesis of a complete EMTM model from the studied components is most often performed based on the principle of superposition. This approach involves simple summation of influences from each component. However, it does not take into account that the components themselves are nonlinear dynamic systems and can actively interact with each other “within” EMTM itself. Moreover, the connections between the components can also be nonlinear. The problem of forming a general EMTM model is further complicated by the need to consider several control actions from the operator or automation systems on individual machine components. In fact, this corresponds to the actual operation of EMTM, when simultaneous control of the running gear, transmission, and working equipment drive occurs.

Analysis of scientific and technical information shows that combining the analytical method and the principle of superposition significantly simplifies the dynamic model of EMTMs and, at the same time, provides satisfactory results for modeling the movement of the machine and its individual elements [1; 2]. The greatest successes have been achieved in the analysis of two design cases:

- determination of the limiting levels of loads acting in the system. In this case, the movement of the EMTM model is considered over short time intervals corresponding to the first quarter of the oscillation period of the mechanical system. Conventionally, such problems are referred to as the dynamics of small displacements;
- calculation of averaged EMTM performance indicators, such as productivity, power balance, etc. In this case, the movement of the system is considered over time intervals compatible in duration with the EMTM work cycle. Such problems are referred to as the dynamics of large displacements.

Each of the listed approaches is characterized by its typical lists of simplifications and assumptions that are used to form the EMTM design model.

Unfortunately, most EMTM models synthesized using the methods considered cannot explain certain facts recorded during EMTM operation. For example, during the operation of wheeled EMTMs, unpredictable and uncontrolled oscillatory processes can occur, both in the machine itself and in its individual systems and mechanisms. This has often been recorded during experiments on real machines at the test site of the Department of Construction and Road Machines of the Kharkiv National Automobile and Highway University. The development of such processes occurs within time intervals lying between “short” intervals corresponding to the dynamics of small displacements, and “long” ones corresponding to the dynamics of large displacements. To prevent such destructive phenomena, additional theoretical and experimental research is necessary.

One solution to the indicated problem, in our opinion, is the development of a generalized EMTM model that allows, during synthesis, to consider not only the features of each specific component, but also its nonlinear connections with all other machine components.

The purpose of the proposed article is to substantiate and develop a generalized dynamic model of a wheeled EMTM, describing the machine motion during the execution of technological operations.

Traditionally, dynamic models are used for the analytical description of EMTM behaviour during technological operations. To obtain an adequate model, it is necessary to use two complementary approaches:

- Structural analysis of movements performed by EMTMs during operation;
- Structural analysis of the machine construction.

Almost all types of EMTMs are characterized by the simultaneous execution of several forced movements. On the one hand, the machine must move around the work site during operation; on the other hand, during this movement, the working element is also simultaneously moved relative to the machine itself. It can be argued that the set of actions performed by EMTMs during operation will be determined by the type of technological operation and the method of its implementation. Thus, structurally, the dynamic model of EMTM should include not only the model of the machine itself, but also dynamic models of all actuators (Fig. 2). In turn, based on the structural analysis of the construction of a specific EMTM, taking into account the principles of system dynamics [3], dynamic models of both the machine as a whole and its individual mechanisms can be synthesized. For a correct description of the space of dynamic states of such a complex nonlinear system, its mathematical model should include not only the equations of motion of the machine and its mechanisms, but also functions describing the controlling external influences, as well as dependencies that determine the relationship between individual equations of motion.

Structural analysis of the construction of various types of wheeled EMTMs revealed common components for which corresponding dynamic models should be developed. These components include: the machine itself, as a set of interconnected moving bodies; the transmission and traction running gear, the steering system, and the working equipment control system (Fig. 2). The working equipment control system can include different actuators, for each of which it is necessary to develop its own dynamic model. Figure 2 uses the following letter designations:

- M_∂, ω_∂ – torque and angular velocity transmitted from the power plant to the corresponding component;
- u – function of external control action on the corresponding component.

Results and discussion

Structural analysis of typical wheeled EMTM designs revealed the commonality of their dynamic schemes (Fig. 3). Most of them are articulated base chassis (masses m_1 and m_2). The working equipment (mass m_3) is articulated to one of the half-frames. The dynamic schemes are nonlinear spatial systems. The position of the system at any time t is described by q_1, q_2, ... q_j, ... q_s coordinates. In the theory of EMTM dynamics, the systems shown in Fig. 3 are considered as holonomic [4; 5].

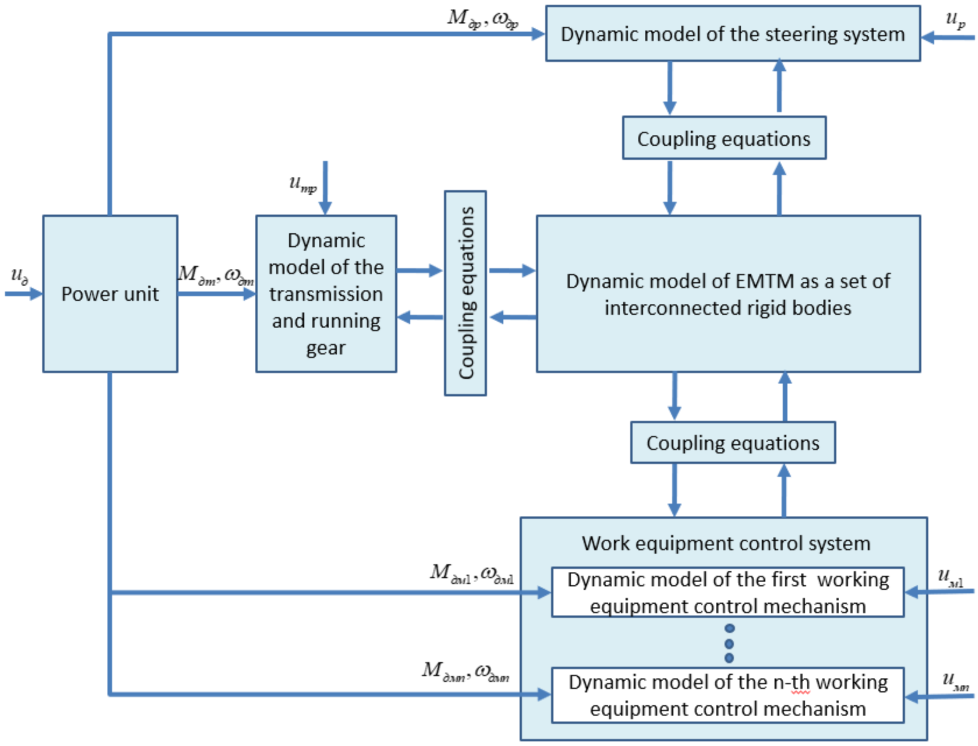

Fig. 2. **Structure of the generalized dynamic model of EMTM**

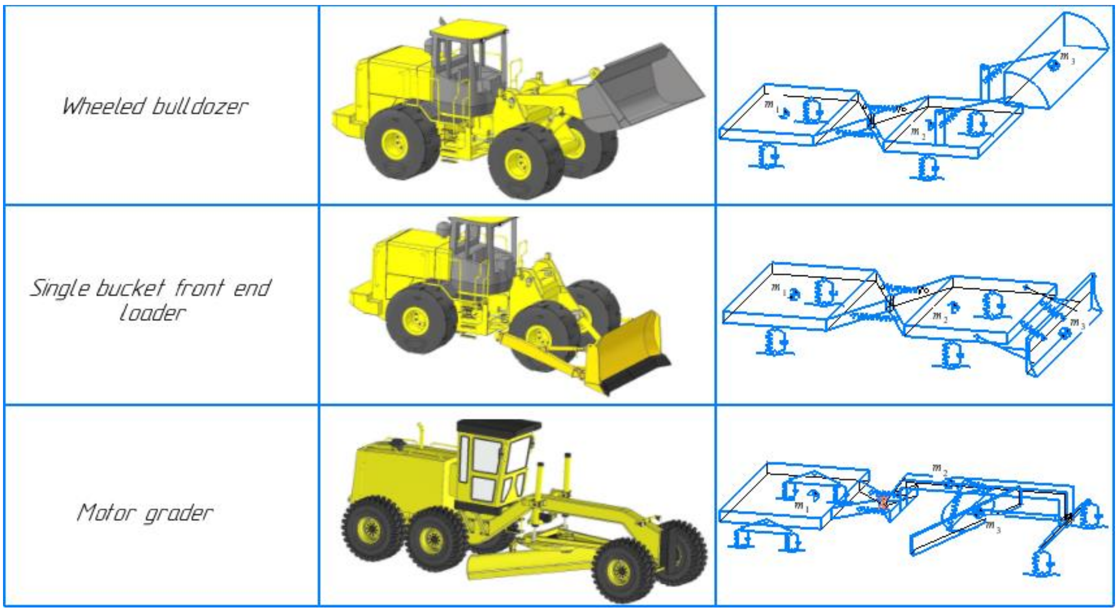

Fig. 3. **Three-dimensional dynamic schemes of EMTM**

The behaviour of EMTM on the worksite depends on a large number of factors. Preliminary analysis allows their division into several groups: Γ – a set of physico-mechanical characteristics of the developed material; Θ – a set of parameters characterizing the support surface; N – a set of parameters characterizing the power plant; M – a set of mass characteristics of the dynamic system; L – a set of geometric parameters characterizing EMTM and its working equipment; C – a set of parameters characterizing the elastic connections in the dynamic system; Λ – a set of parameters characterizing the dissipative connections in the dynamic system. A separate group is the set U, which determines the control actions on the dynamic control systems of the transmission, running gear (steering system) and working equipment. To describe the motion of these three-dimensional dynamic models, it is advisable to use the Lagrange equations of the second kind. The mathematical model of EMTM motion in this case is determined by a system of differential equations of the form

$$\frac{d}{dt}\left\{\frac{\partial\left[K\left(\mathrm{M},\mathrm{L},\dot{q},q,t\right)\right]}{\partial\dot{q}_j}\right\}-\frac{\partial\left[K\left(\mathrm{M},\mathrm{L},\dot{q},q,t\right)\right]}{\partial q_j}= \\ =Q_{Fj}\left(\mathrm{N},\mathrm{L},\Theta,\dot{q},q,t\right)-\frac{\partial\left[\Pi\left(\mathrm{M},\mathrm{L},\mathrm{C},q,t\right)\right]}{\partial q_j}-Q_{Rj}\left(\Gamma,\Lambda,\mathrm{L},\Theta,\dot{q},q,t\right), \tag{1}$$

where $K\left(\mathrm{M},\mathrm{L},\dot{q},q,t\right)$ – kinetic energy of the dynamic system;
$\Pi\left(\mathrm{M},\mathrm{L},\mathrm{C},q,t\right)$ – potential energy of the dynamic system;
$Q_{Fj}\left(\mathrm{N},\mathrm{L},\Theta,\dot{q},q,t\right)$ – driving force acting along the j-th generalized coordinate;
$Q_{Rj}\left(\Gamma,\Lambda,\mathrm{L},\Theta,\dot{q},q,t\right)$ – dissipative force acting along the j-th generalized coordinate.

A feature of the generalized dynamic EMTM model is that the set M can change its characteristics during technological operations. This occurs for the following reasons.

1. As a result of digging the developed material, its mass "joins" the working equipment and begins to move with it. If EMTM is equipped with a blade working body, then as the material is dug, a prism of this material moved by the machine will form in front of the blade, the mass of which can reach 30% ÷ 40% of the EMTM mass. The total moving mass of EMTM in this case will be determined by the dependence:

$$m_{\Sigma}=\begin{cases} m \quad if \quad sign\left(q_x\right)=-1 \\ m+\frac{\rho_{м}}{k_p}\int\limits_{q_{x0}}^{q_x}F\left(h,q_x\right)dq_x \quad if \quad sign\left(q_x\right)=+1 \end{cases}, \tag{2}$$

where m – EMTM mass;
q_x – generalized coordinate coinciding with the longitudinal axis of EMTM;
$\rho_{м}$ – density of the developed material in its natural state;
k_p – material loosening coefficient;
$F(h,q_x)$ – cross-sectional area of the cut layer of material.

EMTM with a bucket working body also moves the material contained in the bucket. The total mass of the machine is determined by the dependence:

$$m_{\Sigma}=m+k_{з}\frac{\rho_{м}}{k_p}V, \tag{3}$$

where k_3 – the bucket filling coefficient, that can vary within the range of 0÷1.3 at a nominal bucket capacity V.

2. During the execution of technological operations, the EMTM working equipment control system changes its position relative to the base chassis. These movements lead to changes in the coordinates of the center of mass of both the working equipment itself and the entire EMTM.

Both of these features allow us to classify EMTM as a class of dynamic systems with variable link mass, which imposes certain limitations on the generalized dynamic model. In particular, studies show that the analytical form of the system motion equations (1) remains unchanged, but during the solution of the system of differential equations, it is necessary to change the values of masses and coordinates of the centers of mass at each step of the numerical calculation in accordance with the performed technological operation.

The variety of transmission and running gear types used in EMTM (Earth-Moving and Transporting Machines) necessitates the implementation of a modular principle for developing the dynamic model of this system. Figure 4 illustrates the dynamic schemes of various transmission types: mechanical, hydrodynamic (with fluid coupling and with torque converter), and hydrostatic.

The basis of the dynamic model of the transmission and running gear is the external characteristic of the power system (ICE) $M_d = f(\omega_d)$ and the characteristics of the joint operation of the engine and the primary transmission unit (fluid coupling, torque converter, hydrostatic transmission) $M_m = f(\omega_m)$. Depending on the type of the primary unit, the form of the graphs of the torque M_m versus the angular velocity ω_m of the output link varies.

The energy from ICE is transmitted to the driving wheels of EMTM sequentially through transmission elements, which are characterized by their inertia I and stiffness C indicators. The mathematical model of the motion of such a system is constructed on the basis of the kinetostatic method and d'Alembert's principle. In general, the mathematical model of the motion of the considered dynamic system has the form:

$$\|I\|\{\ddot{\varphi}\} + \|C\|\{\varphi\} = \{M(t)\}, \tag{4}$$

where $\|I\|$ – inertia coefficient matrix;

$\{\ddot{\varphi}\}$ – generalized acceleration matrix;

$\|C\|$ – stiffness matrix;

$\{\varphi\}$ – generalized coordinate matrix;

$\{M(t)\}$ – generalized disturbing force matrix.

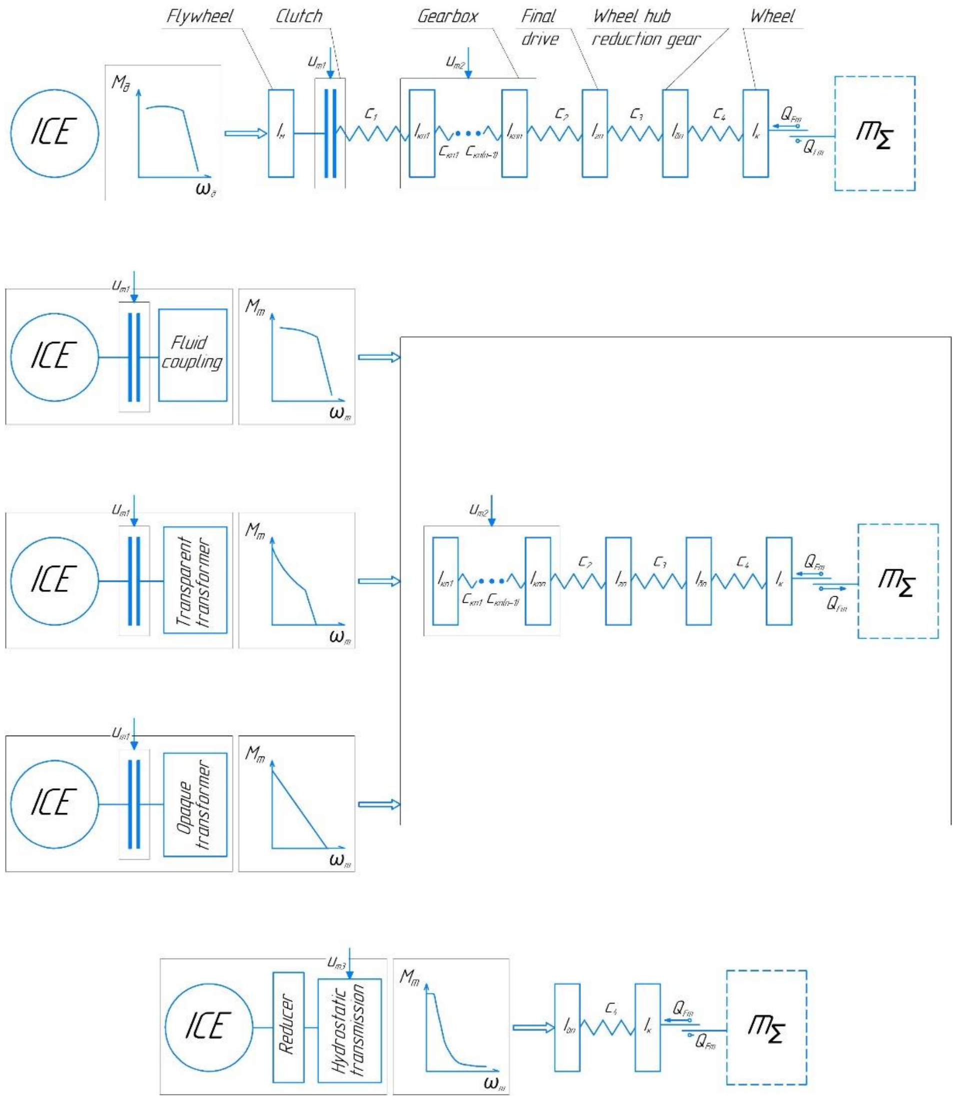

Fig. 4. **Dynamic schemes of the transmission and running gear**

1. the equations relating the dynamic model of the transmission and running gear to the dynamic model of EMTM as a set of interconnected rigid bodies are:

$$Q_{Fm} = m_{\Sigma}\ddot{q}_x sign(q_x) + \Sigma W + \Sigma W_f \ , \tag{5}$$

where ΣW – projection of the principal vector of resistances from the developed medium, projected onto the coordinate axis coinciding with the direction of the EMTM motion;
ΣW_f – total rolling resistance of EMTM;

2. the functions of the time variation of the support reactions on the driving wheels $R_i = f_i(t)$;
3. a function describing the parameters of the dissipative coupling between the two considered dynamic systems;

$$\frac{\dot{\varphi}_\kappa r - \dot{q}_x}{\dot{\varphi}_\kappa r} = A\frac{Q_{Fm}}{R} + B\left(\frac{Q_{Fm}}{R}\right)^n , \tag{6}$$

where $\dot{\varphi}_\kappa$ – angular velocity of the driving wheel rotation;
r – power radius of the wheel;
A, B, n – empirical coefficients.

To control the working equipment of EMTM, a group of hydraulic actuators is introduced into the machine design. Hydraulic cylinders are most often used as the power link [6; 7].

During the movement of the hydraulic cylinder piston, the working body is displaced, which causes a change in the geometric and mass parameters in the main dynamic scheme of EMTM, which is a set of interconnected rigid bodies. This allows synthesizing the coupling equation between the hydraulic drive and the main dynamic model of EMTM as a function of the hydraulic cylinder length [9].

On the other hand, when the overall length of the hydraulic cylinder changes, the working body control mechanism itself alters its geometric and kinematic parameters, which allows it to be classified as a system with variable link masses [10; 11]. This fact must be taken into account when developing the corresponding dynamic model.

A generalized scheme of the hydraulic control mechanism for the EMTM working equipment is shown in Figure 5.

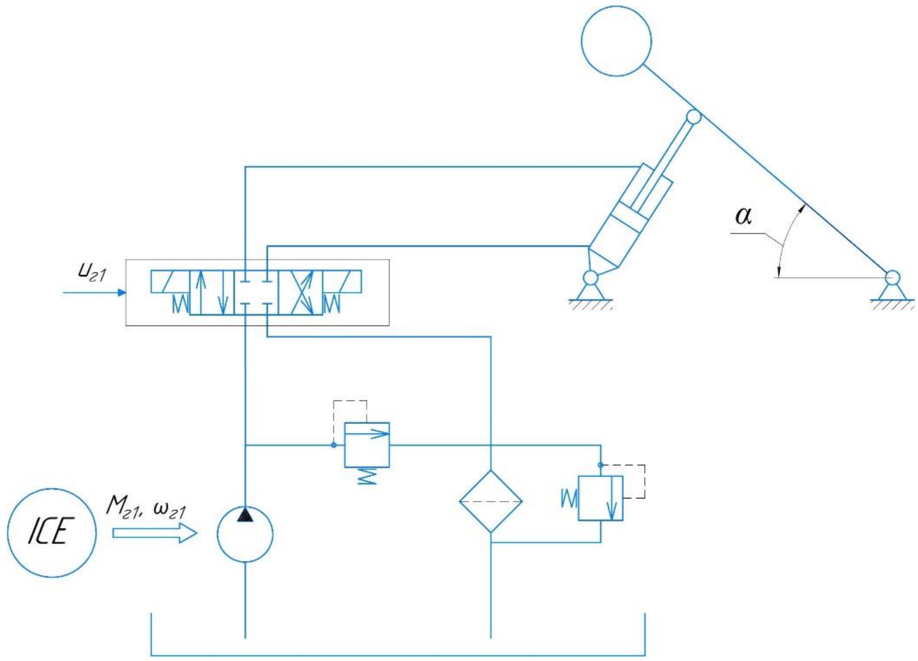

Fig. 5. **Generalized scheme of the hydraulic control mechanism for EMTM working equipment**

We transform this scheme into an equivalent dynamic scheme, Figure 6.

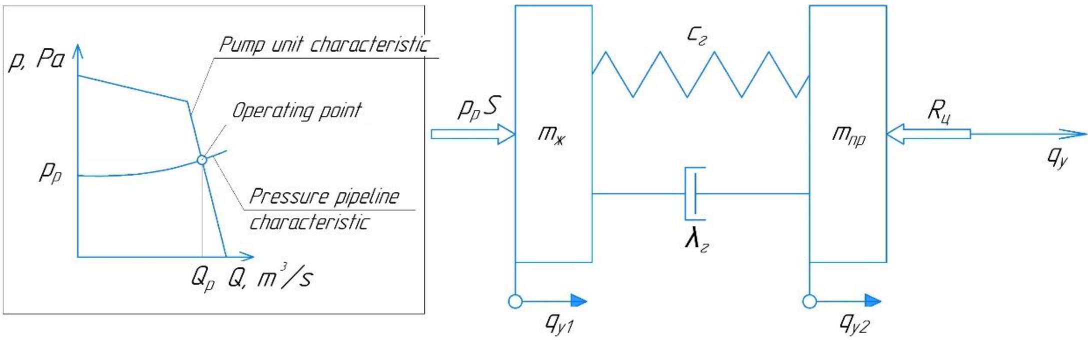

Fig. 6. **Dynamic scheme of the hydraulic drive**

The main performance indicators of the pumping unit are the working pressure p_p and the flow rate Q_p of the hydraulic fluid. Their values are determined by the total pressure losses in the pipelines, the axial load acting on the hydraulic cylinder, and the pump characteristic (Figure 6). We synthesize the mathematical model of the motion of the hydraulic drive elements using the kinetostatic method:

$$\begin{cases} m_{жс}\ddot{q}_{y1} = p_p S - \lambda_г\left(\dot{q}_{y1} - \dot{q}_{y2}\right) - c_г\left(q_{y1} - q_{y2}\right) \\ m_{np}\ddot{q}_{y2} = \lambda_г\left(\dot{q}_{y1} - \dot{q}_{y2}\right) + c_г\left(q_{y1} - q_{y2}\right) - R_ц \end{cases}, \tag{7}$$

where $m_{жс}$ – mass of the moving elements of the hydraulic drive reduced to the hydraulic cylinder rod;
m_{np} – mass of the working equipment with the mass of the developed material attached to it, reduced to the hydraulic cylinder rod;
$\lambda_г$ – dissipation coefficient of the hydraulic system;
$c_г$ – stiffness coefficient of the hydraulic system;
q_y – coordinate axis coinciding with the longitudinal axis of the hydraulic cylinder;
$R_ц$ – total external force acting on the hydraulic cylinder from the working equipment.

The force $R_ц$ defines the coupling equation between the dynamic model of the hydraulic drive and the dynamic model of EMTM as a set of interconnected rigid bodies.

$$R_ц = \Sigma G_y + \Sigma W_y + m_{np}\ddot{q}_y sign(\ddot{q}_y), \tag{8}$$

where ΣG_y – projection of the total gravity vector of the working equipment and the developed material attached to it onto the longitudinal axis of the hydraulic cylinder;
ΣW_y – projection of the total resistance forces from the developed material onto the longitudinal axis of the hydraulic cylinder.

Since the working equipment is part of the EMTM metal structure and moves along the work site with it, when determining, it is also necessary to consider the projection of the working equipment's inertia force onto the longitudinal axis of the hydraulic cylinder – $m_{np}\ddot{q}_y sign(\ddot{q}_y)$.

In the calculations, it is necessary to consider the fact that the values of ΣG_y and m_{np} can vary over a wide range.

The generalized dynamic model is supplemented by a power balance equation, which limits the level of its consumption to the nominal level of the power unit

$$N_д \geq N_m + \Sigma N_г + N_p, \tag{9}$$

where $N_д$ $N_д$ – nominal power of the EMTM power unit;
N_m N_m – power consumed to drive the transmission and running gear;
$\Sigma N_г$ – total power consumed to drive the working equipment control;
N_p – power consumed to drive the steering control.

The structure of the proposed generalized dynamic model of EMTM significantly depends on the control actions, which are functions of time. In general, these actions form a set

$$U = \left[u_{m1}(t), u_{m2}(t), u_{m3}(t), u_{z1}(t), u_{z2}(t)\ldots\right].$$

A significant part of these functions has a step-like character, and their activation is interpreted as the connection of an additional dynamic model of a particular mechanism to the main dynamic model of EMTM. For the correct use of the proposed generalized dynamic model of EMTM, it is recommended to preliminarily create a timing diagram for the activation and deactivation of the transmission and hydraulic drive control mechanisms of the working equipment.

A time diagram represents a coordinate axis of the model time *t*, on which events characterizing control actions on individual mechanisms and systems of EMTM are plotted. For example, if the actuator is a hydraulic cylinder controlled by a hydraulic distributor, the hydraulic system can be in one of three states: the hydraulic cylinder rod is stationary (in a locked state), the rod extends, and the rod retracts. The transition from one state to another will be determined by the event of switching the hydraulic distributor on or off (Fig. 7). For EMTM equipped with a mechanical gearbox, the event axis for the transmission will be determined by the moments of engaging or disengaging a particular gear (Fig. 7).

During the execution of a technological operation, the mechanisms and systems of EMTM are switched on and off independently of each other, and these moments (events) are marked as points C_m on the model time axis *t*. The segments of the model time C_m - C_{m+1} reflect the structure of the dynamic model of EMTM at a given moment in time. The phase coordinates of the system at the final point C_{m+1} of each time segment are the initial phase coordinates for the dynamic model within the time segment C_{m+1} - C_{m+2}.

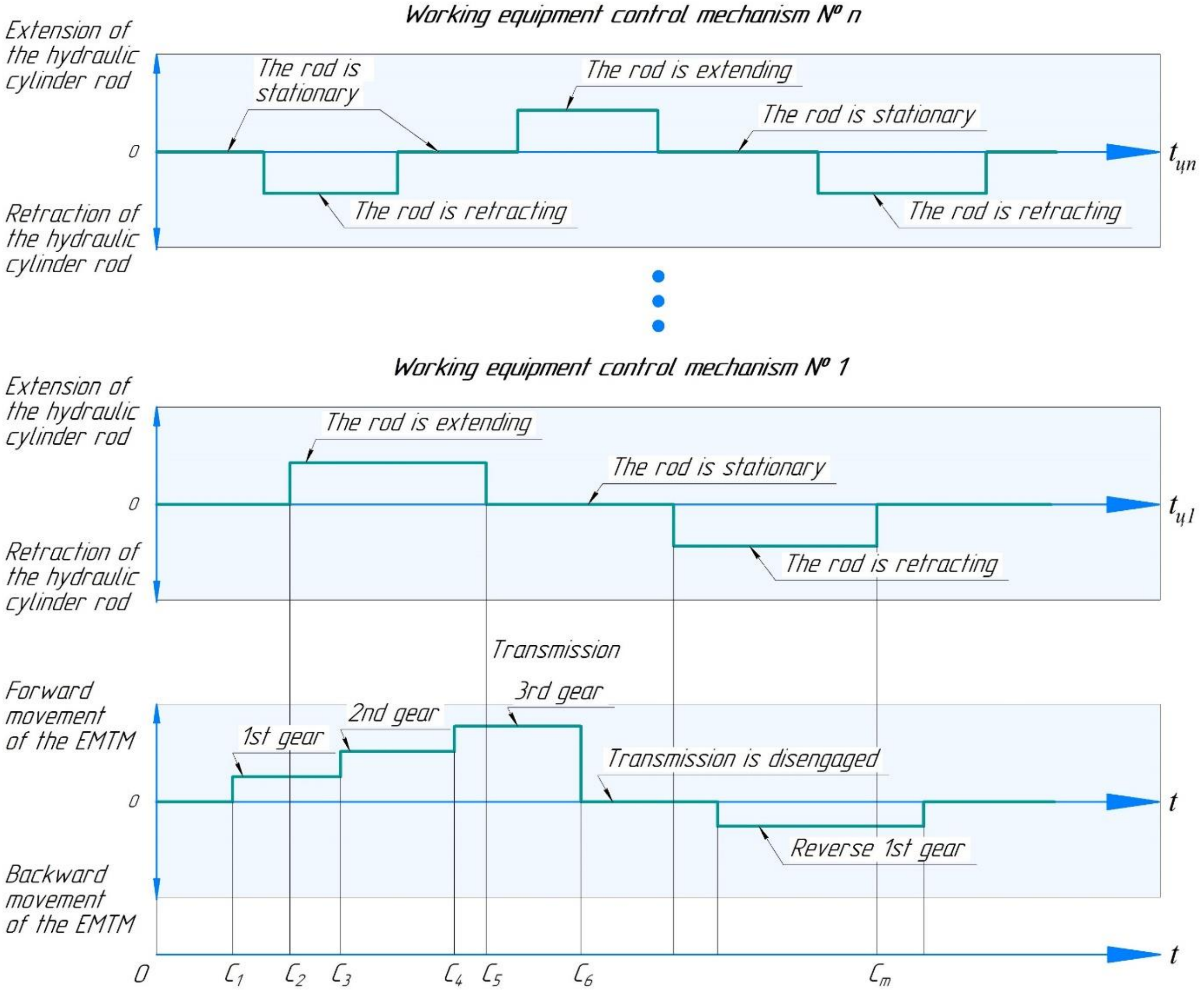

Fig. 7. **Time diagram of the generalized dynamic model of EMTM**

The time diagram confirms the fact that during the execution of a technological operation, the structure of the dynamic model can change due to the appearance and disappearance of parallel dynamic processes simultaneously occurring in the system.

As an example of the formation of a dynamic model of EMTM, let us consider the process of performing a soil digging technological operation by a wheeled bulldozer. Soil cutting is performed with a chip of constant thickness. Two mechanisms participate in the machine operation: the bulldozer transmission together with the running gear, and the hydraulic mechanism for raising and lowering the blade. The blade is controlled by two parallel-mounted hydraulic cylinders. The time diagram of the specified technological process is shown in Fig. 8.

On the model time axis t, four main sections can be distinguished:

- $0 – C_1$ – the 1st transmission gear is engaged, and the bulldozer moves forward with the blade raised. The hydraulic cylinders are in a locked position;
- $C_1 – C_2$ – at time C_1, the hydraulic actuator is engaged, and the hydraulic cylinder rods begin to extend. The bulldozer moves forward, and the hydraulic cylinder rods extend until the blade edge contacts the surface of the soil being excavated;
- $C_2 – C_3$ – the blade penetrates the soil to the cutting depth h. The bulldozer moves forward, and soil digging is performed with a variable thickness of the cut chip;
- $C_3 – C_4$ – the operation of the hydraulic cylinders is stopped, and the rods are locked. The bulldozer moves forward, and soil digging is performed with a constant thickness of the cut chip.

On each of these time intervals, the dynamic model of the bulldozer is described by different systems of motion equations (Table 1).

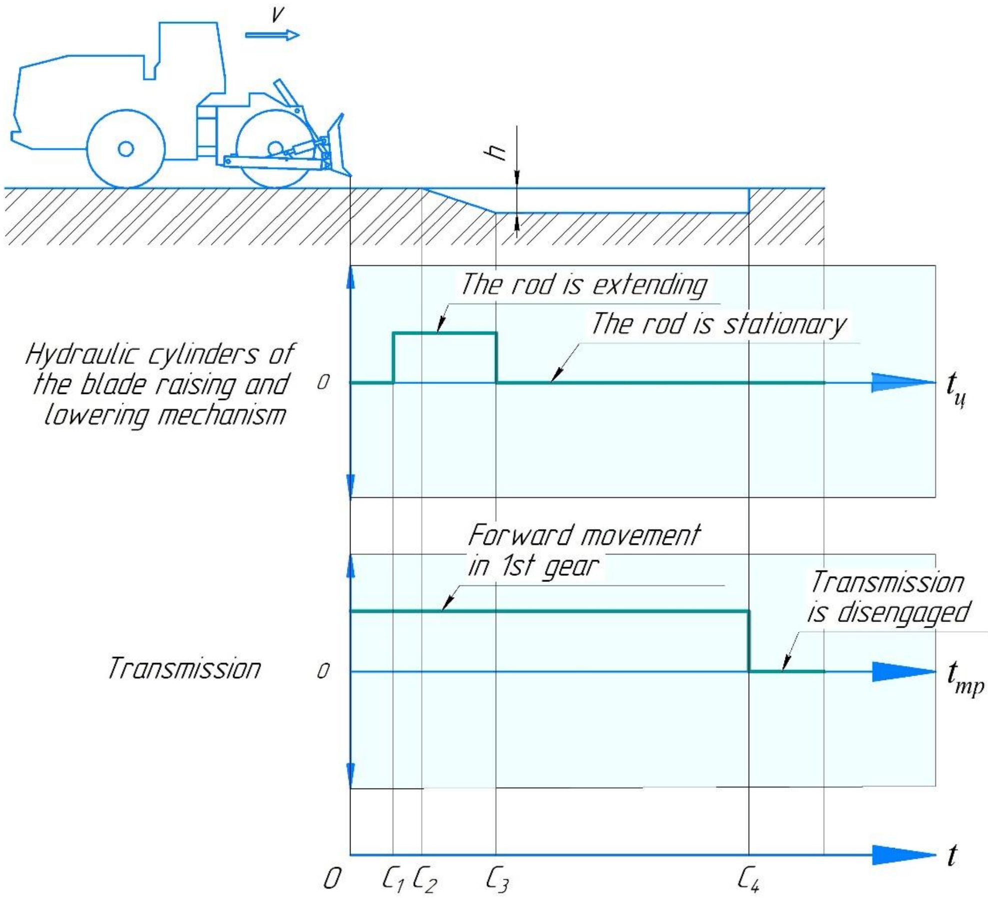

Fig. 8. **Time diagram of the soil digging process with a wheeled bulldozer**

Thus, on each of the four time intervals, the dynamic model of the wheeled bulldozer will be described by different systems of differential equations. For problems formulated in Cauchy form, the initial conditions are equal to the final values of the phase coordinates at the end of each time segment.

Table 1

Resultant dynamic model of a wheeled bulldozer for constant thickness chip soil digging

EMTM Element	Time Diagram Segment			
	$0-C_1$	C_1-C_2	C_2-C_3	C_3-C_4
Dynamic model of the bulldozer as a collection of interconnected rigid bodies	$\frac{d}{dt}\left\{\frac{\partial\left[K\left(\mathrm{M},\mathrm{L},\dot{q},q,t\right)\right]}{\partial\dot{q}_j}\right\}-\frac{\partial\left[K\left(\mathrm{M},\mathrm{L},\dot{q},q,t\right)\right]}{\partial q_j}=Q_{Fj}\left(\mathrm{N},\mathrm{L},\Theta,\dot{q},q,t\right)-$ $-\frac{\partial\left[\Pi\left(\mathrm{M},\mathrm{L},\mathrm{C},q,t\right)\right]}{\partial q_j}-Q_{Rj}\left(\Gamma,\Lambda,\mathrm{L},\Theta,\dot{q},q,t\right),$			
Dynamic model of the transmission and undercarriage	$\lVert I\rVert\{\ddot{\varphi}\}+\lVert C\rVert\{\varphi\}=\{M(t)\}$			
Coupling of equations	$Q_{Fm}=m_{\Sigma}\ddot{q}_x sign(q_x)+\Sigma W+\Sigma W_f$ $\frac{\dot{\varphi}_к r-\dot{q}_x}{\dot{\varphi}_к r}=A\frac{Q_{Fm}}{R}+B\left(\frac{Q_{Fm}}{R}\right)^n$			
Dynamic model of the blade hydraulic lowering mechanism	–	$\begin{cases} m_{ж}\ddot{q}_{у1}=p_p S-\lambda_г\left(\dot{q}_{у1}-\dot{q}_{у2}\right)-c_г\left(q_{у1}-q_{у2}\right) \\ m_{пр}\ddot{q}_{у2}=\lambda_г\left(\dot{q}_{у1}-\dot{q}_{у2}\right)+c_г\left(q_{у1}-q_{у2}\right)-R_ц \end{cases},$		–
		$R_ц=0$	$R_ц=f(t)$	

Conclusions

1. The proposed generalized dynamic model of EMTM allows to take into account the simultaneous operation of several mechanisms and systems that ensure the machine performs technological operations on the work site;
2. For a correct analytical study, the mathematical model of EMTM motion should include
   - a timing diagram of the set of control actions;
   - systems of differential equations (1), (4), (7);
   - equations of coupling between the main dynamic model of EMTM and dynamic models of individual systems and mechanisms;
   - power balance equation (9);
3. The proposed generalized dynamic model of EMTM allows, based on the structural analysis of the machine design, to carry out a variant selection of the transmission dynamic model, which expands the range of its use.

Author contributions

Conceptualization, V.S., methodology, V.S. and O.O., software, I.P., validation, V.S. and O.O., formal analysis, V.S. and O.O., investigation, V.S., O.O., I.P., and O.Y., data curation, O.O. and I.P., writing – original draft preparation, V.S., writing – review and editing, V.S. and O.O., visualization, I.P. and O.Y. All authors have read and agreed to the published version of the manuscript.

References

[1] Timothy W. Dell. Heavy Equipment Power Trains and Systems. AED Foundation, 2018. 992 p.

[2] Хмара Л.А, Кравець С.В., Нічке. В.В., Назаров Л.В., Скоблюк М.П., Нікітін В.Г. Машини для земляних робіт: Навчальний посібник. Рівне-Дніпропетровськ-Харків, 2010, 576 с.

[3] Charles M. Close, Dean K. Frederick, Jonathan C. Newel. Modeling and analysis of dynamic systems. Third Edition. John Wiley & Sons, 2002, 593p

[4] Mitrev R., Janošević D., Marinković D. Dynamical modelling of hydraulic excavator considered as a multibody system. Dinamičko modeliranje hidrauličnog bagera kao sustava sastavljenog od više tijela, Tehnički vjesnik 24, Suppl. 2, 2017, Croatia, pp.327-338.
[5] Helgesson J. Optimization of bucket design for underground loaders. Master's Thesis in the International Master's programme Product Development. Department of Product & Production Development. Chalmers university of technology, Göteborg, Sweden, 2010. 35 p.
[6] El Baji O., Ben Said Amrani N., Sarsri D. Online trajectory tracking control based on the explicit form of the equations of motion for serial manipulator using the new formulation. Journal of Robotics. Volume 2022, Article ID 8715161, Hindawi, 2022, 14 p.
[7] Luengo O., Singh S., Cannon H. Modeling and identification of soil-tool interaction in automated excavation. In Proceedings, IEEE/RSJ International Conference on Intelligent Robotic Systems, October 13-17, 1998 Victoria, B.C., Canada.
[8] Marinković Z., Marinković D., Petrović G., Milić P. Modeling and simulation of dynamic behavior of electric motor driven mechanisms. Technical Gazette. 19, 4(2012), pp. 717-725.
[9] Lin G.C., Wang D.M., Xu L.J., Gao S. The analytical dynamic model of six-DOF industrial robotic manipulators of containing closed chain. Mechanism and Machine Theory 40, 4(2005), pp. 385-393.
[10] Janssen B., Nievelstein M. Multi-loop Linkage Dynamics via Geometric Methods: A Case Study on a RH200 Hydraulic Excavator. Technische Universiteit Eindhoven, Eindhoven, 2005.
[11] Mitrev R., Janošević D., Marinković D. dynamical modelling of hydraulic excavator considered as a multibody system. Technical Gazette 24, Suppl. 2(2017), pp. 327-338